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ANALYSIS OF COLBURN'S ARITHMETICS. IV

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The appearance of Warren Colburn's arithmetics (*First Lessons* and *Sequel*), marked the beginning of a new epoch in the development of arithmetic and arithmetic teaching in the United States. The principal features of these texts may be classed under three heads: (1) the introduction of oral instruction, (2) his ideas about the subject-matter of arithmetic and its organization, and (3) his recognition of sound educational principles. It is the purpose of the writer to present evidence of these features from Colburn's writings.

ORAL INSTRUCTION

To appreciate the significance of Warren Colburn's introduction of oral instruction into arithmetic teaching, it is necessary to recount briefly the method of instruction prevailing before his time. This method may be characterized as the "Cyphering Book" method. In the earlier times rarely did either the teacher or pupil possess a text. Each pupil was provided with a blankbook made of a quire of paper folded and sewed together. The teacher either dictated or set "sums" which the pupil worked on scraps of paper. After the work had been approved by the teacher it was carefully copied in the "Cyphering Book." Such rules as the pupil needed were given by the teacher. Thus the arithmetic work was written as well as being deductive.

Later, when arithmetic texts began to be used in the schools, the method of instruction was not materially changed. For example, in beginning addition the pupil was first given the rule and then as the first problem asked to find the sum of several numbers, some of which were often composed of five or six digits. And similarly in other topics, the first problems were such as to require the use of pencil and paper. Since the numbers were so large that they were beyond the comprehension of the pupil, the work must have appeared to him as a mere juggling with written symbols.

In contrast to this, Colburn does not provide for written computations in the *First Lessons*. In fact he does not introduce the number symbols at all in the first third of the book. Thus the pupil is compelled to perform the operation mentally and to give the result orally. Furthermore, the quantities of the problems throughout the book are small enough to bring the number within the comprehension of the pupil and also so small that he may do the problems mentally. It is therefore probable that pupils solved the problems of the *First Lessons* without recourse to written calculation. When there were no "sums" to be done on paper or slate and submitted to the teacher for inspection, it became necessary for the teacher to hear the pupils give an oral solution of the problem. Thus at least in the case of the younger pupils arithmetic teaching came to be largely oral after the appearance of the *First Lessons*.

Oral instruction required the operations to be performed mentally, i.e., without recourse to written symbols, and the method of oral instruction is probably most widely known as "mental arithmetic."

This feature is not made prominent in the *Sequel* but close connection is made between "operations performed in the mind" and the "application of figures to these operations."

THE SUBJECT-MATTER OF ARITHMETIC AND ITS ORGANIZATION

In his analysis of the subject-matter of arithmetic, Colburn distinguishes between the processes of arithmetic, which he calls "principles," and the applications of arithmetic, which he designates as "subjects." To him the "principles" mean arithmetic and the applications merely a field for the exercise of these principles. Denominate numbers, mensuration, percentage, interest, etc., are not taken as the basis for separate chapters, or even distinct topics. "To give the learner a knowledge of the principles" is his purpose, and to this end the problems are grouped about the principles.

Colburn takes the position that "when the principles are well understood, very few subjects will require a particular rule, and if the pupil is properly introduced to them, he will understand them better without a rule than with one." This is to say, for example, that if a pupil understands well the relation between a product and its factors in all its phases, percentage and its applications require no particular rule and will present no difficulty to the learner. At most, the learner will need to be told the meaning of the new terms used in expressing the problem.

As we would naturally expect from such a point of view, the applications of arithmetic do not influence nor determine the structure of Colburn's texts. To give an idea of the content of the texts and the organization of the subject-matter, the section headings of the *Sequel* are given below.

I. Numeration and notation.
II. Addition.
III. Multiplication, when the multiplier is a single figure.
IV. Compound numbers, factors, and multiplication, when the multiplier is a compound number.
V. Multiplication, when the multiplier is 10, 100, 1000, etc.
VI. Multiplication, when the multiplier is 20, 300, etc.
VII. Multiplication, when the multiplier consists of any number of figures.
VIII. Subtraction.
IX. Division, to find how many times one number is contained in another.
X. Division. Explanation of fractions. Their notation. What is to be done with the remainder after division.
XI. Division, when the divisor is 10, 100, etc.
XII. To find what part of one number another is, or to find the ratio of one number to another.
XIII. To change an improper fraction to a whole or mixed number.
XIV. To change a whole or mixed number to an improper fraction.
XV. To multiply a fraction by a whole number, by multiplying the numerator.
XVI. Division, to divide a number into parts. To multiply a whole number by a fraction.
XVII. To divide a fraction by a whole number. To multiply a fraction by a fraction.
XVIII. To multiply a fraction by dividing the denominator. Two ways to multiply, and two ways to divide a fraction.
XIX. Addition and subtraction of fractions. To reduce them to a common denominator. To reduce them to lower terms.
XX. Contractions in division.
XXI. How to find the divisors of numbers. To find the greatest common divisor of two or more numbers. To reduce fractions to their lowest terms.
XXII. To find the least common multiple of two or more numbers. To reduce fractions to the least common denominator.
XXIII. To divide a whole number by a fraction, or a fraction by a fraction, when the purpose is to find how many times the divisor is con-

tained in the dividend. To find the ratio of a fraction and a whole number, or of two fractions.

XXIV. To divide a whole number by a fraction, or a fraction by a fraction; a part of a number being given to find the whole. This is on the same principle as that of dividing a number into parts.

XXV. Decimal fractions. Numeration and notation of them.

XXVI. Addition and subtraction of decimals. To change a common fraction to a decimal.

XXVII. Multiplication of decimals.

XXVIII. Division of decimals.

XXIX. Circulating decimals. Proof of multiplication and division by casting out 9's.

Even the subdivisions and order of the "principles" are unusual. Multiplication of integers follows addition instead of subtraction. In fractions, multiplication is placed first and is followed by addition and subtraction. Both multiplication and division of fractions are divided into several cases.

The *Sequel* is divided into two parts. The first consists of graded lists of problems with an occasional suggestive note to define some new term or to interpret the meaning of the problem. "The second part contains a development of the principles" based upon problems.

> The two parts are to be studied together, when the pupil is old enough to comprehend the second part by reading it himself. When he has performed all the examples in an article in the first part, he should be required to recite the corresponding article in the second part, not verbatim, but to give a good account of the reasoning. When the principle is well understood, the rules[1] which are printed in italics should be committed to memory.

The Table of Contents of the *Sequel* makes no mention of any of the applications of arithmetic, several of which usually have a chapter devoted to them. However, one must remember that it is Colburn's plan to teach the principles through their use in solving problems. He says:

> As the purpose is to give the learner a knowledge of the principles, it is necessary to have the variety of examples under each principle as great as possible. The usual method of arrangement, according to subjects, has been on this account entirely rejected, and the arrangement has been made according to principles. Many different subjects come under the same principle; and different parts of the same subject frequently come under different principles.

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[1] Few rules are given and such as are given are placed at the end of a section. The rule is the result of a process of development. Colburn gives rules only for the principles.

Colburn mentions the following "subjects" as being specifically included in the text: Compound Multiplication, Addition, Subtraction, and Division; Simple Interest, Commission, Insurance, Duties, and Premiums, Common Discount, Compound Interest, Discount, Barter, Loss and Gain, Simple Fellowship, Compound Fellowship, Equation of Payments, Alligation Medial, Alligation Alternate, Square and Cubic Measure, Duodecimals, Taxes, Mensuration, Geographical and Astronomical Questions, Exchange, Tables of Denominate Numbers.

Colburn omits some topics entirely. He specifically mentions the Rule of Three, Position, and Powers and Roots. The reasons he gives for their omission are:

> Those who understand the principles sufficiently to comprehend the nature of the rule of three, can do much better without it than with it, for when used, it obscures, rather than illustrates the subject to which it is applied.
>
> . . . . This (the rule of Position) is an artificial rule, the principle of which cannot be well understood without the aid of Algebra; and when Algebra is understood, Position is useless. Besides, all the examples which can be performed by Position, may be performed much more easily, and in a manner perfectly intelligible, without it.
>
> Powers and Roots, though arithmetical operations, come more properly within the province of Algebra.

It is interesting to note that some of the omissions which Colburn made nearly a century ago are still considered by some individuals to be debatable.

Not only in his arrangement of topics but also in the very subject-matter itself does Colburn differ radically from the writers of arithmetic who preceded him. In Daboll's *Schoolmaster's Assistant,* which was probably more extensively used in the United States after 1800 than any other arithmetic before Colburn's, the pupil was introduced to the subject as follows:

> Arithmetic is the art of computing by numbers, and has five principal rules for its operations, viz., Numeration, Addition, Subtraction, Multiplication, and Division.
>
> NUMERATION
>
> Numeration is the art of numbering. It teaches to express the value of any proposed number by the following characters, or figures:
>
> 1, 2, 3, 4, 5, 6, 7, 8, 9, 0—or cypher.
>
> Besides the simple value of figures, each has a local value, which depends upon the place it stands in, viz., any figure in the place of units, represents

only its simplest value, or so many ones, but in the second place, or place of tens, it becomes so many tens, or ten times its simple value. . . . .

In the *Scholar's Arithmetic* by Daniel Adams, which was another widely used book in Colburn's time, the subject is begun as follows:

> Arithmetic is the art or science which treats of numbers.
>
> It is of two kinds, theoretical and practical.
>
> The theory of arithmetic explains the nature and quality of numbers, and demonstrates the reason of practical operations. Considered in this sense, arithmetic is a science.
>
> Practical arithmetic shows the method of working by numbers, so as to be most useful and expeditious for business. In this sense arithmetic is an art.

There are six pages of definitions of this sort and an explanation of the system of notation before any problems are given.

Contrast these with the way Colburn introduces the pupil to arithmetic:

FROM "FIRST LESSONS"[1]

1. How many thumbs have you on your right hand? how many on your left? how many on both together?
2. How many hands have you?
3. If you have two nuts in one hand, and one in the other, how many have you in both?
4. How many fingers have you on one hand?
5. If you count the thumb with the fingers, how many will it make?
6. If you shut your thumb and one finger, and leave the rest open, how many will be open?
7. If you have two cents in one hand, and two in the other, how many have you in both?
8. James has two apples, and William has three; if James gives his apples to William, how many will William have?

FROM "SEQUEL"[2]

1. James has 3 cents and Charles has 5; how many have both?
2. Charles bought 3 buns for 16 cents, a quart of cherries for 8 cents, and 2 oranges for 12 cents; how many cents did he lay out?
3. A man bought a hat for 8 dollars, a coat for 27 dollars, a pair of boots for 5 dollars, and a vest for 7 dollars; how many dollars did the whole come to?

In Adams' *Scholar's Arithmetic* division is begun as follows:

> Simple division teaches, having two numbers given of the same denomination, to find how many times one of the given numbers contains the other.

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[1] This and all other citations in this article from the *First Lessons* refer to an edition of 1847.

[2] This and all other citations from the *Sequel* refer to the edition of 1826.

Thus, it may be required to know how many times 21 contains 7; the answer is 3 times. The larger number (21) or number to be divided, is called the *Dividend;* the lesser number (7) or number to divide by, is called the *Divisor;* and the answer obtained (3) the *Quotient.*

After the operation, should there be anything left of the dividend, it is called the *Remainder.* This part, however, is uncertain; sometimes there is no remainder. When it does happen it will always be less than the divisor, if the work be right, and of the same name with the dividend.

RULE

1. "Assume as many figures on the left hand of the dividend as contain the divisor once or oftener; find how many times they contain it, and place the answer as the highest figure of the quotient.
2. "Multiply the divisor by the figure you have found, and place the product under the part of the dividend from which it was obtained.
3. "Subtract the product from the figures above it.
4. "Bring down the next figure of the dividend to the remainder and divide the number it makes up as before."

When you have brought down a figure to the remainder, if the number it makes up be still less than the divisor, a cypher must be placed in the quotient, and another figure brought down.

One example, "Divide 127 by 5," is worked out and explained. Two other problems with divisors of two digits are worked out but not explained. In all there are 11 problems in the list, all abstract.

Colburn begins division in the *First Lessons* by asking, "How many apples, at one cent apiece, can you buy for four cents?"

Twenty-three problems of this sort are given before any attempt is made to consider division as a process. Then it is first considered in a practical problem, and immediately following, the same combination is called for in an abstract problem.

These quotations are typical of the texts of Daboll and Adams. They are also illustrative of the prevailing attitude toward the subject-matter of arithmetic before Colburn's texts appeared. It is very evident that he had a different notion of the subject-matter of arithmetic.

Colburn invariably introduces a topic or a new combination by a "practical question." If the topic or combination is new, the "practical question" is followed by the same problem in abstract form. For example, the following is taken from p. 94 of the *First Lessons:*

1. If a yard of cloth cost 3 dollars, what will 1 half of a yard cost?
2. What is 1 half of 3?

3. If a barrel of beer cost 5 dollars, what will 1 half of a barrel cost?
4. What is 1 half of 5?
5. If 2 barrels of cider cost 7 dollars, what is that a barrel?
6. What is 1 half of 7?
7. What is 1 half of 9?
8. What is 1 half of 11?

One phase of the organization of the subject-matter is Colburn's treatment of the symbols of notation which seems to exemplify one of his fundamental notions of arithmetic. For example, he wishes the pupil to learn that two objects and one object make a total of three objects; that five plums and four plums are nine plums, and *not* that the symbols 2+1 equal the symbol 3, or the symbols 5+4 equal the symbol 9. As a means to this end, in the *First Lessons*, the characters 1, 2, 3, ... are not given until p. 50 and the system of notation and numeration is not given beyond 10 until p. 69. Before these symbols and the system of notation and numeration are given, the pupil has learned the four fundamental operations for integers. The symbols are introduced by saying, "Instead of writing the names of numbers, it is usual to express them by particular characters called figures." Thus before the pupil is asked to learn the number symbols, he doubtless has felt the need for them. For this reason he is more likely to appreciate their meaning and usefulness.

Another feature of Colburn's texts is that, so far as I am aware, they represent the first explicit attempt to write a series of arithmetic texts. The *Sequel* is a continuation of the *First Lessons* and yet begins with the first of arithmetic. The *Sequel* is the equivalent in scope and type of problems to the other arithmetics of that time, but the *First Lessons* is a pioneer in a new field. The author intended it to be especially adapted to young children. It is stated in the edition of 1847 that "almost any child of five or six years is capable of understanding more than half the book, and those of seven or eight years old can understand the whole of it."

[*To be continued*]